MATTERS OF GRAVITY

Number 2 Fall 1993

Table of Contents

Editorial

Jorge Pullin
Center for Gravitational Physics and Geometry
Penn State University

Welcome to the second issue of *Matters of Gravity*, a newsletter for the gravitational physics community of the United States.

Two years have passed since the appearance of the first issue. In the meantime, Peter Saulson has stepped down as editor and I took over recently to try to restore continuity to the Newsletter. I would try to assure that we get published at least twice a year, once in the fall and in the spring. Publication dates are September and February 1st.

As before, the intention of the newsletter is to try to bring a sense of community to the people working in gravitational physics in the US. Everyone is welcome to contribute about any topic dealing with gravitaional physics. Articles are not to exceed two pages in length and should avoid talking about one's own work. In this issue we incorporate a new format of articles: reports about conferences. Conference organizers and anyone else interested in writing such reports are welcome to contribute.

Putting together a newsletter like this implies a certain non negligible amount of effort. It would be healthier for the newsletter if we had more than one editor. This would also help strike some balance in the topics covered. It would be extremely helpful if we had an editor with experimental interests. Anyone willing to invest some effort is welcome to join in as editor.

As before, we keep a group of correspondents to try to get input from as wide an audience as possible. Many thanks to them for their cooperation in putting this issue together. As in the previous issue, we invite anyone who wants to serve as correspondant to join in.

This issue has been distributed in two forms. A paper edition, produced in TeX, was mailed to a list we derived mainly from the NSF's list of sponsored workers in gravitational physics. A larger number of you are receiving *Matters of Gravity* via e-mail. Due to technical problems in PSU we sent the Newsletter out on Malcolm MacCallum's list and posted it in the LANL gr-qc bulletin board. Since the Newsletter is focused on the US community, we should plan to have a more focused distribution mechanism for the future. The newsletter is also available as a postscript file for those interested.

I would like to end by thanking the contributors. Writing these short articles requires more effort than people may think.

Please also send me your comments on this second issue of the newsletter, as well as suggestions for articles.

Jorge Pullin

Center for Gravitational Physics and Geometry
The Pennsylvania State University
University Park, PA 16802-6300

Fax: (814)863-9608
Phone (814)863-9597
Internet: pullin@phys.psu.edu

Correspondents

1. John Friedman and Kip Thorne: Relativistic Astrophysics,
2. Jim Hartle: Quantum Cosmology and Related Topoics
3. Gary Horowitz: Interface with Mathematical High Energy Physics, including String Theory
4. Richard Isaacson: News from NSF
5. Richard Matzner: Numerical Relativity
6. Abhay Ashtekar and Ted Newman: Mathematical Relativity
7. Bernie Schutz: News From Europe
8. Lee Smolin: Quantum Gravity
9. Cliff Will: Confrontation of Theory with Experiment
10. Peter Bender: Space Experiments
11. Riley Newman: Laboratory Experiments
12. Peter Michelson: Resonant Mass Gravitational Wave Detectors
13. Robbie Vogt: LIGO Project

Some recent work in general relativistic astrophysics

John Friedman, University of Wisconsin - Milwaukee

Stephen Detweiler has been studying the evolution of a binary black hole system by examining solutions of the Einstein equations which are periodic when viewed from some rotating frame of reference. The approach generalizes an earlier study with Blackburn of solutions stationary in a rotating frame. When the fractional energy loss per period is small, the radiation field near the binary system is small; and the Detweiler-Blackburn solutions with equal amounts of ingoing and outgoing radiation can approximate the evolution of a binary black hole system from the time when the holes are far apart, through the stage of slow evolution caused by gravitational radiation reaction, up until the time when the radiation reaction timescale is comparable to the dynamical timescale.

Detweiler's approximation allows one to use a variational principle to estimate key features of a binary system. Accurate estimates are claimed for the relationship between the total energy and angular momentum of the system, the masses and angular momenta of the components, the rotational frequency of the frame of reference in which the system is periodic, the frequency of the periodicity of the system, the stationary mass and current moments and the amplitude and phase of each multipole component of gravitational radiation.

*

The study of nonradial oscillations of relativistic stars, begun 25 years ago by Thorne and his colleagues, has recently been renewed in work by Chandrasekhar and Ferrari (For a review, see V. Ferrari, "Non-radial oscillations of stars in general relativity: a scattering problem," in Classical General Relativity, ed. S. Chandrasekhar, Oxford, 1993). Chandrasekar and Ferrari discuss the oscillations as a problem of resonant scattering of gravitational waves incident on a potential barrier generated by the spacetime curvature. The scattering viewpoint is required for axial perturbations, which do not couple to the star and have no newtonian counterpart; but even for polar perturbations, a decoupling of matter and metric perturbations allows one to view the perturbations as a scattering problem. Although the polar perturbations are reducible to a Schrödinger equation only outside the star, Chandrasekhar and Ferrari find in the interior a set of equations that involve only the perturbed metric. Once the system has been solved, the motion of the fluid can be obtained algebraically. A conserved current expresses a flux of gravitational "energy" through a perturbed star in a way that generalizes the Regge theory of resonant scattering to the present context. The local energy flux is gauge-dependent, but the total power radiated is not.

New classes of outgoing modes (which they call resonances) have emerged from this work. Outgoing axial modes can occur in usually dense spherical models, while in rotating stars, the coupling of polar and axial perturbations results in a family of rotationally induced outgoing modes.

*

New models of rapidly rotating neutron stars have been constructed recently by Cook, Shapiro and Teukolsky (Rapidly Rotating Neutron Stars in General Relativity: Realistic Equations of State, Cornell preprint CRSR 1047), and by Eriguchi, Hachisu and Nomoto (Proc. Roy. Astr. Soc., in press). Both groups use codes based on a code developed initially for relativistic polytropes by Komatsu, Eriguchi and Hachisu. The Cook et. al. work presents sequences of stars with fixed baryon number, which can be taken to represent the evolution of star that is losing angular momentum. Rotation supports stars with masses above the maximum allowed for a spherical star, and Cook et. al. highlight a feature of rotating sequences that was not noticed in previous models considered by Friedman, Ipser and Parker. As a neutron star loses angular momentum, it ordinarily spins down, but near the lower limit on angular momentum, the angular velocity increases as the angular momentum decreases, and they suggest that the unexpected spin-up may provide an observable precursor to collapse.

*

Ipser and Lindblom (*The Astrophysical Journal*, **389**, 392 (1992)) have begun the difficult work needed to generalize to relativistic stars the elegant formalism they used in newtonian gravity to compute normal modes of rotating stars. For short wavelength modes, one can use the "Cowling approximation," neglecting the change in the gravitational potential. The relativistic equations of motion then have essentially the same character as the perturbed Euler equations, and the hydrodynamic degrees of freedom of the adiabatic pulsations of relativistic fluids can be described by a single scalar potential. This potential is determined by a second-order (typically elliptic) partial differential equation. They obtain a variational principle from which the pulsation frequencies may be evaluated in this approximation, and which can be generalized to an Eulerian principle that does not require the Cowling approximation.

Work by Mendel and Lindblom ("Superfluid Effects on the Stability of Rotating Newtonian Stars," in *The Structure and Evolution of Neutron Stars*, ed. by D. Pines, R. Tamagaki and S. Tsuruta (Addison-Wesley: 1992) pp. 224-226) on dissipation in rotating superfluids shows an unexpectedly large internal dissipation in rotating neutron stars. The result apparently implies stablility against nonaxisymmetric modes in neutron stars cooler than the superfluid transition temperature of about 10^9 K. Fast pulsars with weak enough magnetic fields would then be limited in their angular velocity only by the Kepler frequency, the angular velocity of an particle in orbit just outside the equator.

Two Dimensional Black Holes

Gary T. Horowitz, UCSB

There has been a great deal of interest in the past couple of years on the subject of two dimensional black holes. One might well ask, "Why is there so much interest in a subject clearly removed from physical applications?". The answer is two-fold. Since two dimensions is clearly simpler than four, one can use these black holes as toy models to study processes which cannot yet be analyzed in higher dimensions. In addition, there are arguments which show that certain properties of extremal four dimensional black holes are accurately described by the two dimensional solution.

There are actually two quite different lines of research which are being pursued. The first is to try to understand the endpoint of Hawking evaporation of two dimensional black holes, and to answer the question of whether pure states evolve to mixed states. An important step in this direction was taken by Callan, Giddings, Harvey, and Strominger (Phys. Rev. 45 (1992) R1005). They study black holes in a theory of a metric and scalar field (the dilaton) in two spacetime dimensions. This theory can be quantized exactly using canonical quantization, but it has no local degrees of freedom and hence no Hawking radiation. If one adds a scalar field, then it turns out that one can find exact solutions describing the classical collapse of matter to form a black hole. One can then calculate the Hawking radiation in this classical background using the trace anomaly (as originally pointed out many years ago by Christensen and Fulling). One sees explicitly how the radiation "turns on" and approaches a constant rate at late times.

Perhaps the most important advantage of working in two dimensions is that one can include the backreaction of the radiation on the black hole. However, even in two dimensions, it appears difficult to compute all quantum effects exactly. If one considers a large number N of scalar fields, one can calculate an effective action to leading order in $1/N$. Extrema of this effective action then represent the evolution of a black hole including backreaction. The quantum corrected equations are too complicated to solve exactly, but numerical calculations show that the black hole evaporates in a finite time. Unfortunately, when the black hole becomes very small, the $1/N$ approximation breaks down. In other words, the situation is similar to the four dimensional case: One has to go beyond the $1/N$ or semiclassical approximation to understand the endpoint of Hawking evaporation. However, it has been pointed out that there are many inequivalent quantizations of this classical theory. In some of them the quantum theory is exactly soluble. In their simplest form, the soluble theories appear to be unphysical since the energy is unbounded from below, and black holes continue to radiate indefinitely. However, a modification has been proposed (corresponding to a boundary condition at strong coupling) which cures this problem. It is currently a matter of debate whether this modified theory preserves quantum mechanical predictability.

The second line of research involving the black hole concerns its connection to string theory. The equations of motion for the metric and dilaton (which are solved by the 2D black hole) are part of the low energy field equations for string theory. The exact string equations include extra terms involving higher powers of the curvature. Witten showed how to construct an exact solution to string theory which reduces to the black hole in the low energy limit (E. Witten, Phys. Rev. D 44 (1991) 314). This was the first first time that an exact black hole solution was found. (More recently, exact black holes have also been found in three and four dimensions, although these examples are not asymptotically flat.) People are now studying various aspects of this exact solution. Perhaps the most important is the existence of a singularity. Singularities in string theory should be defined by studying the motion of test strings. There are now several examples of geodesically incomplete spacetimes, including some with curvature singularities, for which the motion of test strings is well behaved. In other words, string theory "resolves" certain curvature singularities classically, without including quantum effects of spacetime. (See, e.g. J. Horne, G. Horowitz and A. Steif, Phys. Rev. Lett. 68 (1992) 568.) The exact 2D black hole spacetime has a curvature singularity, and the current indications are that it *is* singular in string theory. So the quantum gravity effects will be important. Preliminary work is underway to study the evaporation of the black hole in the full context of string theory.

Resonant-Mass Gravitational Wave Detectors - An Update

Peter Michelson, Stanford

Currently, two cryogenic resonant-mass detectors, operating at 4 K, are in continuous operation at Louisiana State University and at CERN (University of Rome group). The LSU detector has been in nearly continuous operation for more than two years with an rms sensitivity of $h = 6 \times 10^{-19}$. They report less than one non-Gaussian event per day. Coincidence analysis of data from the last half of 1991 was carried out with the Rome group and a new upper limit on the g.w. flux impinging on the earth was reported at the last international conference in Argentina. This is an improvement on the previous upper limit reported from a three-way coincidence with detectors at LSU, CERN, and Stanford.

At Stanford, work is continuing on the assembly of a new ultralow temperature detector designed to operate at 50 mK. Fabrication of the major components of the detector have been completed. Prof. H.J. Paik's group at the University of Maryland is closely collaborating with the Stanford group and are supplying a resonant- mass superconducting inductive transducer for the Stanford detector. The cryostat for the new system has successfully undergone a series of leak tests from room temperature to 77 K. It is expected that the antenna will be installed in the cryostat by the end of this year.

The Nautilus ultralow temperature detector of the Rome group has been installed at the INFN laboratory in Frascati. Frascati is now the center of activity for the Rome group. The INFN has provided excellent new facilities for the gravity wave detection effort there, including a new building. An effort of similar magnitude is underway at the INFN Laboratory in Legnaro, Italy. The group there, headed by Massimo Cerdonio, recently sponsored a Workshop on Resonant-Mass Detectors. The meeting was attended by about 100 people representing nearly every active group in the field. There was extensive discussion of currently operating detectors, progress on ultralow temperature detectors, and on the feasibility of constructing a new generation of massive, spherical antennas.

In September, the US groups doing research on resonant-mass detectors will hold a two day meeting to plan a detailed feasibility study on spherical, resonant-mass detectors. Issues about vibration isolation and suspension, cryostat design, and antenna fabrication will be addressed as well as formulation of a plan for coordinating the efforts of the various groups involved in the study.

Universality and Scaling in Gravitational Collapse

Richard Matzner, University of Texas at Austin

Recent results by Matthew Choptuik (Texas) on spherical scalar collapse, and by Andrew Abrahams (Cornell) and Charles Evans (Chapel Hill) on axisymmetric gravitational collapse, show a surprising scaling behavior, and critical behavior similar to that found in many phase transition phenomena.

The work by Choptuik is a high accuracy spherical study, with minimally or nonminimally coupled scalar radiation interacting only gravitationally. He uses a mesh refinement scheme as suggested by Berger and Oliger (1984), and is able to evaluate variations over $\sim 10^7$ in scale by adaptively refining the grid as appropriate. The Abrahams/Evans code uses a simpler type of adaptation which also allows for increased resolution of key regions of the computational domain when needed. It evaluates pure vacuum axisymmetric behavior, imploding shells of gravitational radiation.

By using his adaptive techniques, Choptuik has demonstrated the ability of the algorithm to resolve solution features on essentially arbitrarily small spatial and temporal scales. He has used it to perform an exhaustive survey of the solution space of his model. (Choptuik, 1992; 1993a) In the course of this survey a variety of unanticipated, and apparently generic, features of strongly self-gravitating pulses of massless scalar radiation were discovered. They appear to be the manifestation of "critical" behavior in the model: there are points in parameter-space where the details of the solution exhibit an exponential dependence on the initial (t=0) configuration of the scalar field. The solution wave profile becomes independent of the initial shape, and an echo is observed; repeated transformed copies of a standard waveform emerge. Further, a generic parameter dependence for the formation of black holes is found. Choptuik sets up a sequence of examples of localized spherical imploding waves which differ only in one parameter, say wave amplitude. As the amplitude of the wave approaches the critical value which initiates black hole formation the final outgoing scalar wave profile takes on the characteristic behavior. A series of waveforms is emitted which are of roughly constant amplitude, but of logarithmically self similar shape in space and time. As a particular evolution approaches a limit time, the wave pattern repeats with a time compression of about $e^{3.4}$ per repeat; there is also a logarithmic self similar shape in space. Choptuik can estimate from the numerics both the time and spatial compression factor and finds them closely equal. Because of the extremely fine zoning allowed by his local grid refinement technique, Choptuik is able to determine the value of the parameter (here the amplitude) leading to black hole formation, to machine precision (~ 1 in 10^{13} on a Cray machine). As the amplitude is adjusted to lie just above the critical value a*, a black hole forms, and a remarkable analog of other critical phenomena appears: $m_{BH} = m_f |a - a*|^\gamma$ where m_f is a constant and γ is an exponent numerically determined by Choptuik: $\gamma = 0.37...$. This means that black holes of arbitrarily small mass can form. As the amplitude approaches a* from above, one also sees a logarithmic repetition of outgoing waveforms. What is truly remarkable and particularly characteristic of critical phenomena is that the same scaling holds regardless of the control parameter; for instance, narrowness of the pulse can be used to control the formation or not of the black hole; the same echoing and the same critical exponent are found. Further, this behavior with the same exponent holds for black hole formation from infalling minimally coupled scalar fields (Choptuik 1992, 1993a), and nonminimally coupled scalar fields (Choptuik 1992, 1993b). The extreme dynamic range of Choptuik's results $\sim 10^7$ is such that, even with today's fastest machines, they would have been extremely difficult to obtain without his use of the adaptive algorithm.

Remarkably similar results were obtained by Abrahams and Evans, for gravitational wave implosions to form black holes. (Abrahams 1992, Evans 1992, Abrahams and Evans 1993) Their code is computationally complicated because it is axisymmetric rather than spherical. Although the mesh structure is fixed, grid adaptation is achieved via a grid velocity which provides increased central resolution during the period of strong field dynamics and/or black hole formation. The Abrahams/Evans code explores a very important direction in the physics of this phenomenon since it is nonspherical and vacuum. Evans and Abrahams find very close agreement for the black hole mass behavior, finding the same dependence of the black hole mass on the wave strength, with the same power gamma. They also find an echoing similar to that in Choptuik's scalar case, but with substantially different exponent. Whereas Choptuik's rescaling occurs with a factor $e^{3.4} \sim 30$, Abrahams and Evans find apparently $e^{0.6} \sim 1.8$ for the factor. Better nonspherical collapse codes may eventually be needed to verify the echoing result, but the black hole mass result seems very convincing. The exponent 0.37 for the formed mass of the black hole, incidentally, is solidly in the range of critical exponents for condensed matter phase transitions.

A. Abrahams, in Syracuse Workshop on Numerical Relativity/Black Holes, G. Fox, editor (1992).

A.M. Abrahams and C.R. Evans, "Critical Behavior and Scaling in Vacuum Axisymmetric Gravitational Collapse," Phys. Rev. Lett., 70, p. 2980 (1993).

M.S. Berger and J. Oliger, J. Comp. Phys. 53 p. 484 (1984).

M. Choptuik, "Critical Behavior in Massless Scalar Field Collapse," in Approaches to Numerical Relativity, R. d'Inverno, ed., Cambridge University Press, (1992).

M. Choptuik, "Universality and Scaling in Gravitational Collapse of a Massless Scalar Field," Phys. Rev. Letters, 70, p. 9 (1993a).

M. Choptuik, private communication (1993b).

C.R. Evans, in Syracuse Workshop on Numerical Relativity/Black Holes, G. Fox, editor (1992).

Gravitational Wave Memories Upgraded: The Christodoulou Effect

Richard Price, University of Utah

The physical effect of a gravitational wave is the displacement of free masses by the strain h_{ij} of the wave. If a burst of waves pass, the stationary value of h_{ij} after the passage of the waves will not be the same as that before the waves. In principle this would leave the free masses of a gravitational wave detector in positions different than their positions before the waves arrived, and the shifted positions would constitue a souvenir of the burst. That gravitational (and other) wave trains have such "memory" has long been known, as has the connection with the "zero frequency limit" (ZFL) of the fourier decomposition of the wave amplitude. In practice this ZFL feature is the basis for detection. The waves in the burst need only to be measured at a frequency below the lowest natural frequency in the wave burst (i.e., the reciprocal of the burst duration) [1]. The measurement of the true DC offset of detector masses would be swamped by drift and by low frequency noise.

The reason for recent interest in wave memories is the correction of a widespread and longstanding conceptual error concerning their calculation. It had been accepted that the calculation of the memory produced by an astrophysical event required only linearized gravity theory, even for a strong field events like black hole collisions. The argument went something like this: Only $1/r$ contributions to h_{ij} are radiative; for the memory only the initial and final values of h_{ij} are needed; before and after the strong accelerations producing the fat middle of the burst, the stress energy is simply that of "particles" moving at uniform velocity; the $1/r$ fields of these particles is just the Coulomb fields due to their masses, and those contributions are completely within the scope of linear theory.

Since the memory of a nonlinear interaction could be calculated with linear theory, it seemed to give a cheap partial answer to questions, e.g, about the radiation from a collision of black holes.[2] There were some disquieting unresolved issues. Philip Payne [3], for instance, noticed that the ZFL argument for black holes led to inconsistencies. A nonlinear contribution to the memory was present in the studies of equation of motion by Luc Blanchet and Thibault Damour [4], but its importance escaped notice. All this changed about a year ago when Demetrious Christodoulou, with a careful and mathematically rigorous analysis, showed that the memory does in general have nonlinear contributions [5]. This nonlinear contribution is now called the "Christodoulou part of the memory."

The flaw in the linear-is-adequate analysis is that in the final state, after the passage of the wave burst, the sources of the field are not only the particles in uniform motion; there is also the gravitational radiation of the burst. This turns out to be quantitatively, as well as pictorially, the origin of the Christodoulou contribution. As Kip Thorne has pointed out, the previously believed relationship between memory and the asymptotic states of the interacting particles remains valid if the gravitons of the final state are included[6]. Since the stress-energy of the gravitons generated in a strong field interaction (e.g., black hole collision) can be comparable to that for the initial and final states of the holes, the graviton contribution to the memory cannot be ignored.

This picture helps clarify aspects of the Christodoulou contribution, such as why there is interest in gravitons generating memory (i.e., zero frequency radiation) but not in their generation of waves at other frequencies. The answer lies in the fact that the gravitational wave stress- energy originates in the motion of the stress-energy of the source. Each orbit of, e.g., a binary neutron star will generate an amount of wave energy rather small in comparison with the particle (i.e., neutron star) mass-energy. For each orbit, then, the radiation is a considerably weaker source of waves than the stars themselves. For the memory, however, it is the totality of the energy generated over several or many cycles that competes in strength with the "real" stress-energy of the stars.

Calculations by Alan Wiseman and Clifford Will[7] are in accord with this picture. They considered gravitational bremsstrahlung, the small angle scattering of one mass (hole, star, etc) by another, and find that the Christodoulou contribution to memory must be small. (There is relatively little radiation generated, and only one "cycle.") For coalescing neutron star binaries, the presently favored source for wave bursts, the situation is quite different. The complete waveforms for the coalescence are not known, but Wiseman and Will use a model calculation to estimate that the Christodoulou contribution to the memory may be as much as 20% of the amplitude of the quadrupole radiation.

It has been a bit of an embarrassment for a widely accepted "truth" to be wrong. Kip Thorne has claimed, in print[6], that this defeat of handwaving by mathematics has been a "salubrious experience." Presumably we will all wave our hands a bit more carefully for a while, but the experience might turn out to be useful for more than just building character. The Christodoulou contribution gives a measure of the total wave power generated. For binary neutron star coalescence the total power will depend on the details of the final tidal disruptions, in particular on the radii of the constituent neutron stars. The Christodoulou contribution to the memory of the wavetrain might give us the most precise observable measure of this neutron star physics.[8]

[1] For a discussion of the measurement of wave memory, see V. B. Braginsky and K. S. Thorne, Nature (London) 327, 123 (1987).

[2] L. L. Smarr, Phys. Rev. D 15, 2069 (1977).

[3] P. N. Payne, Phys. Rev. D 28, 1894 (1983).

[4] L. Blanchet and T. Damour, in Proceddings of the 12th International Conference on General Relativity and Gravitation, 1989, Conference Abstracts (unpublished), p. 265. L. Blanchet, Habilitation thesis, Universite Pierre et Marie Curie, Paris, 1990, pp. 205-207.

[5] D. Christodoulou, Phys. Rev. Lett. 67, 1486 (1991).

[6] K. S. Thorne, Phys. Rev. D 45, 520 (1992).

[7] A. G. Wiseman and C. M. Will, Phys. Rev. D 44, R2945(1991).

[8] E. Flanagan, D. Kennefick, D. Markovic, and K. S. Thorne, paper in preparation.

Conference Report:

Topical Conference on Quantum Aspects of Balck Holes

Santa Barbara June 21-27 1993

Jeff Harvey, Gary Horowitz, and Andy Strominger, Coordinators

The topical conference on Quantum Aspects of Black Holes was held from June 21-27 1993 and was attended by 113 physicists, including most of the world's experts on the quantum physics of black holes. The conference followed two separate but related workshops held at the ITP, one entitled *Non-Perturbative String Theory*, and the other *The Small-Scale Structure of Space-Time.*

Different aspects of black holes, string theory, and quantum gravity were discussed with the focus on the question of whether the physics of black holes requires a fundamental change in the laws of quantum mechanics. The possibility of an irreconcilable difference between black hole physics and quantum predictability was raised in 1975 by S. Hawking and emphasized in his presentation at the conference. Hawking argued that black holes decay by emitting a thermal spectrum of particles, thus apparently losing any information about the matter which originally collapsed to form the black hole. This loss of information contradicts quantum mechanical predictability which requires microscopic preservation of information.

Interest in this question has been revived in the last two years by the discovery of simplified models of black holes which allow a more precise treatment of quantum corrections to the process of Hawking evaporation. At the time of the conference those in attendance were split into two definite camps: those believing in loss of information, and those arguing that quantum predictability need not be given up. One of the outcomes of this conference was a fairly detailed set of problems that each camp must address. Roughly speaking, the information loss camp needs to develop a specific set of calculational rules which supplant the usual deterministic rules of quantum mechanics and does not allow unacceptably large violations of energy conservation or causality at low energies. The quantum predictability camp has had difficulty in providing a plausible explanation of how the information is retained in the radiation coming from the black hole and has had to postulate the existence of new microscopic degrees of freedom at the horizon of the black hole. Their task is to provide a specific model of these new degrees of freedom and at least a heuristic description of how they can restore information to the Hawking radiation.

The conference did not resolve this fundamental question, but it did provide a framework for future results, sharpened the problems which need to be addressed, and left participants with new energy and enthusiasm for tackling these problems.

Conference Report:

Workshop on knots and Quantum Gravity

Riverside May 14-16 1993

John Baez, University of California at Riverside

The loop representation is a promising approach to the quantization of gravity, in that it is nonperturbative and manifestly respects the general covariance of the theory. The physics behind it is conservative, essentially amounting to the canonical quantization of Einstein's equations. It is, however, mathematically innovative, and has forged connections between general relativity and the superficially unrelated subject of knot theory. The goal of this workshop was to bring together physicists and topologists and begin a dialog that might catalyze further research on these connections. It took place on May 14th-16th at the University of California, Riverside, under the auspices of the mathematics and physics departments. The proceedings will appear in a volume entitled Knots and Quantum Gravity, to be published by Oxford University Press.

On Friday the 14th, Dana Fine spoke on "Chern-Simons theory and the Wess-Zumino-Witten model." Witten's original work deriving the Jones invariant of links (or, more precisely, the Kauffman bracket) from Chern-Simons theory used conformal field theory as a key tool, and by now the relationship between conformal field theory in 2 dimensions and topological quantum field theories in 3 dimensions has been explored from a number of viewpoints. The path integral approach, however, has not yet been worked out in full mathematical rigor. In this talk Fine described work in progress on reducing the Chern-Simons path integral on S^3 to the path integral for the Wess-Zumino-Witten model.

Also, Oleg Viro spoke on "Simplicial topological quantum field theories." Recently there has been increasing interest in formulating topological quantum field theories (TQFTs) in a manner that relies upon triangulating spacetime. In a sense this is an old idea, going back to the Regge-Ponzano model of Euclidean quantum gravity. However, this idea was given new life by Turaev and Viro, who rigorously constructed the Regge-Ponzano model of 3d quantum gravity as a TQFT based on the $6j$ symbols for the quantum group $SU_q(2)$. Viro discussed a variety of approaches of presenting manifolds as simplicial complexes, cell complexes, etc., and methods for constructing TQFTs in terms of these data.

Saturday's talks began with a thorough review of recent work on the loop representation of quantum gravity by Renate Loll, Abhay Ashtekar and Jorge Pullin. Loll's talk was entitled "Loop formulation of gauge theory and gravity." This served to introduce the loop representation and its various physical applications. Ashtekar's talk, "Loop transforms," largely concerned his new work with Jerzy Lewandowski on making the loop representation into rigorous mathematics. The notion of measures on the space $\mathcal{A}/\mathcal{G}$ of connections modulo gauge transformations has long been a key concept in gauge theory, which however has been notoriously difficult to make precise. The key notion developed by Ashtekar and

Isham for this purpose is the "holonomy C*-algebra," an algebra of observables generated by Wilson loops. Formalizing the notion of a measure on $\mathcal{A}/\mathcal{G}$ as a state on the holonomy C*-algebra, Ashtekar and Lewandowski have been able to construct such a state with remarkable symmetry properties, in some sense the natural analog of Haar measure on a compact Lie group. Ashtekar also discussed the implications of this work for the study of quantum gravity.

Pullin spoke on "The quantum Einstein equations and the Jones polynomial." Perhaps the most remarkable connection between knot theory and quantum gravity is that, using the loop representation, the Jones polynomial invariant of links (or more precisely, the Kauffman bracket) represents a state of quantum gravity with cosmological constant, essentially a quantization of anti-deSitter space. Pullin detailed his work with Bernd Brüegmann and Rodolfo Gambini on this subject. He also sketched the proof of a new result, that the coefficient of the 2nd term of the Alexander-Conway polynomial represents a state of quantum gravity with zero cosmological constant.

On Saturday afternoon, Louis Kauffman spoke on "Vassiliev invariants and the loop states in quantum gravity." One aspect of Brüegmann, Gambini and Pullin's work that is especially of interest to knot theorists is that it involves extending the bracket invariant to generalized links admitting certain kinds of self-intersections. This concept also plays a major role in the study of Vassiliev invariants of knots. Curiously, however, the extensions occuring in the two cases are different. Kauffman explained the relationship between the two from the path-integral viewpoint.

Sunday morning began with a talk by Gerald Johnson, "Introduction to the Feynman integral and Feynman's operational calculus," on his work with Michel Lapidus on rigorous path integral methods. Viktor Ginzburg then spoke on "Vassiliev invariants of knots," and in the afternoon, Paolo Cotta-Ramusino spoke on "4d quantum gravity and knot theory." This talk dealt with his work in progress with Maurizio Martellini. Just as Chern-Simons theory gives a great deal of information on knots in 3 dimensions, there appears to be a relationship between a certain class of 4-dimensional theories and so-called "2-knots," that is, embedded surfaces in 4 dimensions. This class includes quantum gravity in the Ashtekar formulation, Donaldson theory, and the so-called $B \wedge F$ theory. Cotta-Ramusino and Martellini have described a way to construct observables associated to 2-knots, and are endeavoring to prove at a perturbative level that they give 2-knot invariants.

Louis Crane spoke on "Quantum gravity, spin geometry, and categorical physics." This was a review of Crane's work on the use of 2-categories in 4-dimensional quantum gravity and his construction with David Yetter of a 4-dimensional TQFT based on the "15j symbols" for $SU_q(2)$. Just as every 3d TQFT gives rise to a braided tensor 2-category, and thus solutions to the Yang-Baxter equations, Crane's generalization of TQFT suitable for 4 dimensions gives rise to a braided tensor 2-category, and thus solutions to the Zamolodchikov tetrahedron equations. In the conference proceedings there will also appear a paper by J. Scott Carter and Masahico Saito, "Knotted Surfaces, Braid Movies, and Beyond," dealing with their work on 2-knots, 2-braids and 2-tangles. All these topological objects can be drawn as movies in which links evolve in time, with one "elementary string interaction" occuring between each frame. Just as any two pictures of the same knot can be bridged by a sequence of Reidemeister moves, Carter and Saito have developed a set of "movie moves" relating any two movies of the same 2-knot. More recently, they have developed a theory of 2-braids. Their paper discusses these matters and also reviews new solutions of the Zamolodchikov tetrahedron equations.

In the final talk of the workshop, John Baez spoke on "Strings, loops, knots and gauge fields." He attempted to clarify the similarity between the loop representation of a generally covariant gauge theory and a topological string theory. At a fixed time both involve loops or knots in space, but the string-theoretic approach should clarify the role of surfaces embedded in spacetime.

Conference Report:

NATO Advanced Research Workshop on Deterministic Chaos in General Relativity

Kananaskis, Alberta, Canada,26–30 July 1993

Beverly Berger, Oakland University

The Workshop was organized to address a controversy that had arisen in recent years over whether or not the dynamics of the vacuum, diagonal Bianchi IX (Mixmaster) cosmology is chaotic. Although most of the discussion concerned this topic, the focus of the meeting broadened to include other exciting nonlinear phenomena within general relativity.

The workshop began with an overview of the main issues by Hobill. It has long been known that the approach to the singularity of the Mixmaster model can be approximated as an infinite sequence of Kasner epochs. The epoch changes when the trajectory in minisuperspace (MSS) bounces off the MSS potential (spatial scalar curvature). In the anisotropy plain, the trajectory bounces between two walls which form a corner of the MSS potential, moving outward from the origin (isotropy) but with the trajectory angle with the outward pointing corner ray increasing after each bounce. At some point, the trajectory points inward rather than outward so that it moves to a new corner where the process begins again. This represents the start of a new era. In the late 1960's, Belinskii, Lifshitz, and Khalatnikov (BKL) showed that this evolution could be expressed as a discrete map for u which parametrizes each Kasner epoch. Within a single era, the map is $u_{n+1} = u_n - 1$ (for $u_n \geq 2$). At the end of an era, $u_{n+1} = (u_n - 1)^{-1}$ (for $1 \leq u_n \leq 2$). It is only the era change which is sensitive to initial conditions (SIC). The sequence of era changes is the chaotic Gauss map $u_{N+1} = (u_N - [u_N])^{-1}$ (where [ ] denotes integer part and u_N is the u value that starts each era). One measure of chaos is the Liapunov exponent (LE) which measures the divergence of initially nearby trajectories and is positive for chaotic systems. (The Gauss map has a LE > 0. In the late 1980's, it became possible to evaluate LE's for numerical solutions of ordinary differential equations. Application of these techniques to Mixmaster by independent groups (e.g. Burd et al, Hobill et al) yielded the surprising result of vanishing LE implying that the Mixmaster dynamics is not chaotic in direct conflict with the Gauss map result. To further confuse the issue, the Mixmaster LE values depended on the choice of variables used, leading to LE > 0 in some cases (e.g. Pullin). Various analyses purporting to demonstrate Mixmaster to be either chaotic or not chaotic then began to appear in the literature. It became clear that the difficulty with the LE is due to the exponential increase in epoch duration in terms of some of the time coordinates used. However, there remained significant confusion over the exact nature of Mixmaster dynamics and whether or not an appropriate measure could be found to answer the chaos question once and for all.

To provide a framework for the discussion of these questions at the workshop, introductions to chaotic dynamics were given by Churchill and Bishop while MacCallum provided an overview of relativistic cosmologies. Misner reviewed both his original formulation and that of the Misner-Chitre coordinates which allow an approximate treatment of the model as geodesic flow on a space of negative curvature with a MSS potential of fixed location in the relevant anisotropy plane. Coley presented Wainwright's variables (based on the analysis of Bogoyavlensky) which allow unification of the treatment of all Class A Bianchi Types and can be used to show rigorously how Bianchi Types I and II attract the Bianchi IX solution between bounces and during the bounce respectively. Berger used an amalgam of the BKL and Misner approaches to demonstrate the relationship of the BKL approximate discrete evolution to the Mixmaster dynamics. Additional observations on all these approaches were given by Rugh, Burd, and Creighton. The juxtaposition of this variety of treatments demonstrated that Mixmaster dynamics itself is well-understood with SIC at an era change only. This "weak" SIC must be reflected correctly for any measure of chaos in the model to be regarded as successful. Yet the issue of chaos itself may be semantic rather than substantive since the answer given by any measure may depend on the weighting it gives to the regular and SIC parts of the Mixmaster dynamics.

Other solutions in general relativity exhibit interesting nonlinear dynamics. Choptuik discussed the strange echoing wave form and critical scaling he has observed in a numerical study of the collapse of a spherically symmetric scalar field. Berger displayed the competition between the growth of small scale structure and the development of velocity dominance seen numerically in the approach to the singularity of the unpolarized Gowdy T^3 cosmology. Chrusciel applied dynamical systems theory to the properties of Cauchy horizons required by Hawking's Chronology Protection Conjecture. Calzetta discussed the application of a method using homoclinic orbits to test for chaos in both test body trajectories in perturbed black hole spacetimes and in a Friedmann-Robertson-Walker (FRW) model with conformally coupled scalar field. Tavakol used the chaotic trajectories possible in closed negatively curved FRW models (with nontrivial topology) to study the anisotropy of the cosmic microwave background. Ribeiro introduced a fractal cosmology in order to reproduce the observed galaxy-galaxy correlation function. Tomaschitz discussed chaos in the context of quantum fields on curved spacetime.

The 25 participants found the discussion free-ranging and stimulating. The interactions between mathematicians and physicists, between dynamicists and relativists made the workshop particularly worthwhile. The proceedings, edited by Hobill et al, will be published by Plenum.